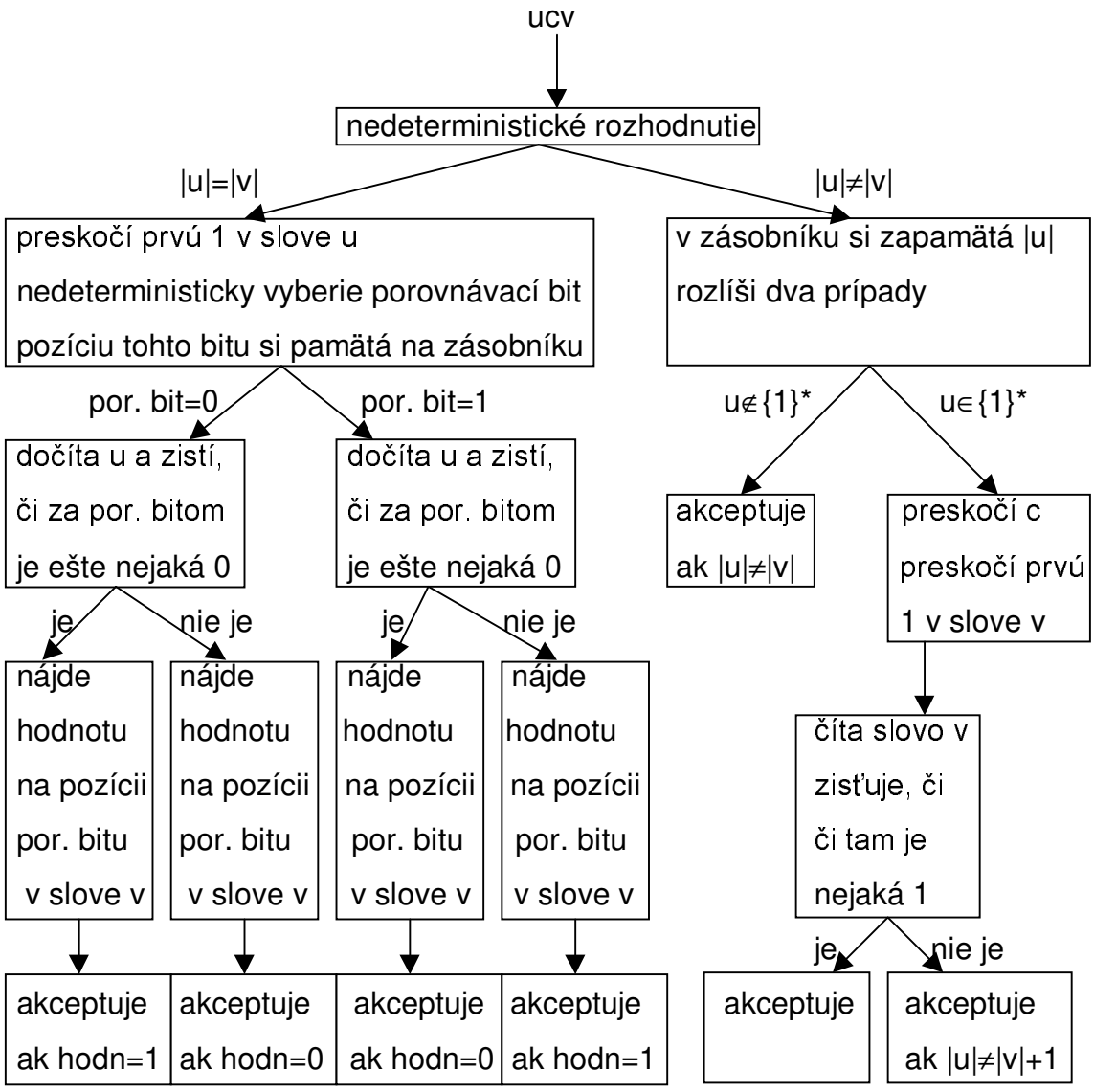
Zadanie:

Zostrojte zásobníkový automat pre jazyk

L = { ucv | u,v∈1{0,1}*; c je terminálny symbol a u,v nie sú binárne zápisy po sebe idúcich čísel takých, že u≤v }

Neformálny popis automatu A:

Budeme konštruovať nedeterministický zásobníkový automat A pre jazyk L. Automat A najskôr popíšeme neformálne pomocou nasledovného diagramu:



Formálny popis automatu A:

Nedeterministický zásobníkový automat A pre jazyk L vyzerá nasledovne:

$A = (K, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

kde: $K = \{ q_0, q_1, q_{10}, q_{1001}, q_{1002}, q_{1003}, q_{1011}, q_{1012}, q_{1013}, q_{1014}, q_{11}, q_{1101}, q_{1102}, q_{1103}, q_{1111}, q_{1112}, q_{1113}, q_{1114}, q_{211}, q_{212}, q_{213}, q_{214}, q_{215}, q_{216}, q_{221}, q_{222}, q_{223}, q_{224}, q_{225}\}$

je konečná množina stavov,

$\Sigma = \{ 0, 1, c \}$ je vstupná abeceda,

$\Gamma = \{ Z_0, 1 \}$ je množina (abeceda) zásobníkových symbolov,

$q_0 \in K$ je počiatočný stav automatu,

$Z_0 \in \Gamma$ je počiatočný symbol na vrchu zásobníka,

$F \subseteq K, F = \{ q_{1003}, q_{1014}, q_{1103}, q_{1114}, q_{214}, q_{215}, q_{216}, q_{223}, q_{224} \}$

je množina akceptačných stavov automatu,

$\delta : K \times (\Sigma \cup \{ \varepsilon \}) \times \Gamma \rightarrow 2_k^{K \times \Gamma^*}$ je prechodová funkcia

($2_k^{K \times \Gamma^*}$ označuje konečné podmnožiny $K \times \Gamma^*$) definovaná nasledovne:

1.) a) $\delta (q_0, 1, Z_0) = \{ (q_1, Z_0) \}$

b) $\delta (q_0, \varepsilon, Z_0) = \{ (q_{211}, Z_0) \}$

2.) a) $\delta (q_1, 0, Z_0) = \{ (q_1, Z_0 1), (q_{10}, Z_0) \}$

b) $\delta (q_1, 1, Z_0) = \{ (q_1, Z_0 1), (q_{11}, Z_0) \}$

c) $\delta (q_1, 0, 1) = \{ (q_1, 11), (q_{10}, 1) \}$

d) $\delta (q_1, 1, 1) = \{ (q_1, 11), (q_{11}, 1) \}$

3.) a) $\delta (q_{10}, 1, Z_0) = \{ (q_{10}, Z_0) \}$

b) $\delta (q_{10}, 1, 1) = \{ (q_{10}, 1) \}$

c) $\delta (q_{10}, c, Z_0) = \{ (q_{1001}, Z_0) \}$

d) $\delta (q_{10}, c, 1) = \{ (q_{1001}, 1) \}$

e) $\delta (q_{10}, 0, Z_0) = \{ (q_{1011}, Z_0) \}$

f) $\delta (q_{10}, 0, 1) = \{ (q_{1011}, 1) \}$

4.) a) $\delta (q_{1001}, 1, Z_0) = \{ (q_{1002}, Z_0) \}$

b) $\delta\ (q_{1001},\ 1,\ 1) = \{\ (q_{1002},\ 1)\ \}$

5.) a) $\delta\ (q_{1002},\ 0,\ 1) = \{\ (q_{1002},\ \varepsilon)\ \}$

b) $\delta\ (q_{1002},\ 1,\ 1) = \{\ (q_{1002},\ \varepsilon)\ \}$

c) $\delta\ (q_{1002},\ 0,\ Z_0) = \{\ (q_{1003},\ Z_0)\ \}$

6.) a) $\delta\ (q_{1003},\ 0,\ Z_0) = \{\ (q_{1003},\ Z_0)\ \}$

b) $\delta\ (q_{1003},\ 1,\ Z_0) = \{\ (q_{1003},\ Z_0)\ \}$

7.) a) $\delta\ (q_{1011},\ 0,\ Z_0) = \{\ (q_{1011},\ Z_0)\ \}$

b) $\delta\ (q_{1011},\ 0,\ 1) = \{\ (q_{1011},\ 1)\ \}$

c) $\delta\ (q_{1011},\ 1,\ Z_0) = \{\ (q_{1011},\ Z_0)\ \}$

d) $\delta\ (q_{1011},\ 1,\ 1) = \{\ (q_{1011},\ 1)\ \}$

e) $\delta\ (q_{1011},\ c,\ Z_0) = \{\ (q_{1012},\ Z_0)\ \}$

f) $\delta\ (q_{1011},\ c,\ 1) = \{\ (q_{1012},\ 1)\ \}$

8.) a) $\delta\ (q_{1012},\ 1,\ Z_0) = \{\ (q_{1013},\ Z_0)\ \}$

b) $\delta\ (q_{1012},\ 1,\ 1) = \{\ (q_{1013},\ 1)\ \}$

9.) a) $\delta\ (q_{1013},\ 0,\ 1) = \{\ (q_{1013},\ \varepsilon)\ \}$

b) $\delta\ (q_{1013},\ 1,\ 1) = \{\ (q_{1013},\ \varepsilon)\ \}$

c) $\delta\ (q_{1013},\ 1,\ Z_0) = \{\ (q_{1014},\ Z_0)\ \}$

10.) a) $\delta\ (q_{1014},\ 0,\ Z_0) = \{\ (q_{1014},\ Z_0)\ \}$

b) $\delta\ (q_{1014},\ 1,\ Z_0) = \{\ (q_{1014},\ Z_0)\ \}$

11.) a) $\delta\ (q_{11},\ 1,\ Z_0) = \{\ (q_{11},\ Z_0)\ \}$

b) $\delta\ (q_{11},\ 1,\ 1) = \{\ (q_{11},\ 1)\ \}$

c) $\delta\ (q_{11},\ c,\ Z_0) = \{\ (q_{1101},\ Z_0)\ \}$

d) $\delta\ (q_{11},\ c,\ 1) = \{\ (q_{1101},\ 1)\ \}$

e) $\delta\ (q_{11},\ 0,\ Z_0) = \{\ (q_{1111},\ Z_0)\ \}$

f) $\delta\ (q_{11},\ 0,\ 1) = \{\ (q_{1111},\ 1)\ \}$

12.) a) $\delta\ (q_{1101}, 1, Z_0) = \{\ (q_{1102}, Z_0)\ \}$

b) $\delta\ (q_{1101}, 1, 1) = \{\ (q_{1102}, 1)\ \}$

13.) a) $\delta\ (q_{1102}, 0, 1) = \{\ (q_{1102}, \varepsilon)\ \}$

b) $\delta\ (q_{1102}, 1, 1) = \{\ (q_{1102}, \varepsilon)\ \}$

c) $\delta\ (q_{1102}, 1, Z_0) = \{\ (q_{1103}, Z_0)\ \}$

14.) a) $\delta\ (q_{1103}, 0, Z_0) = \{\ (q_{1103}, Z_0)\ \}$

b) $\delta\ (q_{1103}, 1, Z_0) = \{\ (q_{1103}, Z_0)\ \}$

15.) a) $\delta\ (q_{1111}, 0, Z_0) = \{\ (q_{1111}, Z_0)\ \}$

b) $\delta\ (q_{1111}, 0, 1) = \{\ (q_{1111}, 1)\ \}$

c) $\delta\ (q_{1111}, 1, Z_0) = \{\ (q_{1111}, Z_0)\ \}$

d) $\delta\ (q_{1111}, 1, 1) = \{\ (q_{1111}, 1)\ \}$

e) $\delta\ (q_{1111}, c, Z_0) = \{\ (q_{1112}, Z_0)\ \}$

f) $\delta\ (q_{1111}, c, 1) = \{\ (q_{1112}, 1)\ \}$

16.) a) $\delta\ (q_{1112}, 1, Z_0) = \{\ (q_{1113}, Z_0)\ \}$

b) $\delta\ (q_{1112}, 1, 1) = \{\ (q_{1113}, 1)\ \}$

17.) a) $\delta\ (q_{1113}, 0, 1) = \{\ (q_{1113}, \varepsilon)\ \}$

b) $\delta\ (q_{1113}, 1, 1) = \{\ (q_{1113}, \varepsilon)\ \}$

c) $\delta\ (q_{1113}, 1, Z_0) = \{\ (q_{1114}, Z_0)\ \}$

18.) a) $\delta\ (q_{1114}, 0, Z_0) = \{\ (q_{1114}, Z_0)\ \}$

b) $\delta\ (q_{1114}, 1, Z_0) = \{\ (q_{1114}, Z_0)\ \}$

19.) a) $\delta\ (q_{211}, 1, Z_0) = \{\ (q_{211}, Z_01)\ \}$

b) $\delta\ (q_{211}, 1, 1) = \{\ (q_{211}, 11)\ \}$

c) $\delta\ (q_{211}, 0, 1) = \{\ (q_{221}, 11)\ \}$

d) $\delta\ (q_{211}, c, 1) = \{\ (q_{212}, 1)\ \}$

20.) a) $\delta\ (q_{212}, 1, 1) = \{\ (q_{213}, 1)\ \}$

21.) a) $\delta\ (q_{213}, 0, 1) = \{\ (q_{213}, \varepsilon)\ \}$

b) $\delta\ (q_{213}, 1, 1) = \{\ (q_{214}, 1)\ \}$

c) $\delta\ (q_{213}, \varepsilon, 1) = \{\ (q_{215}, 1)\ \}$

d) $\delta\ (q_{213}, 0, Z_0) = \{\ (q_{216}, Z_0)\ \}$

e) $\delta\ (q_{213}, 1, Z_0) = \{\ (q_{216}, Z_0)\ \}$

22.) a) $\delta\ (q_{214}, 0, 1) = \{\ (q_{214}, 1)\ \}$

b) $\delta\ (q_{214}, 1, 1) = \{\ (q_{214}, 1)\ \}$

23.) a) $\delta\ (q_{216}, 0, Z_0) = \{\ (q_{216}, Z_0)\ \}$

b) $\delta\ (q_{216}, 1, Z_0) = \{\ (q_{216}, Z_0)\ \}$

24.) b) $\delta\ (q_{221}, 0, 1) = \{\ (q_{221}, 11)\ \}$

c) $\delta\ (q_{221}, 1, 1) = \{\ (q_{221}, 11)\ \}$

d) $\delta\ (q_{221}, c, 1) = \{\ (q_{225}, 1)\ \}$

25.) a) $\delta\ (q_{225}, 1, 1) = \{\ (q_{222}, \varepsilon)\ \}$

26.) a) $\delta\ (q_{222}, 0, 1) = \{\ (q_{222}, \varepsilon)\ \}$

b) $\delta\ (q_{222}, 1, 1) = \{\ (q_{222}, \varepsilon)\ \}$

c) $\delta\ (q_{222}, \varepsilon, 1) = \{\ (q_{223}, 1)\ \}$

d) $\delta\ (q_{222}, 0, Z_0) = \{\ (q_{224}, Z_0)\ \}$

e) $\delta\ (q_{222}, 1, Z_0) = \{\ (q_{224}, Z_0)\ \}$

27.) a) $\delta\ (q_{224}, 0, Z_0) = \{\ (q_{224}, Z_0)\ \}$

b) $\delta\ (q_{224}, 1, Z_0) = \{\ (q_{224}, Z_0)\ \}$

Dôkaz správnosti konštrukcie automatu A - L(A) = L

Najskôr si dokážeme pomocné tvrdenie, ktoré nám samotný dôkaz L(A)=L zjednoduší:

Lema: Nech $u,v \in \{0,1\}^*$, $u=a_1...a_n$, $v=b_1...b_n$, $a_1= b_1=1$, $n \leq 2$, nech $i \in \{1..n\}$ také, že $a_i=0$. Nech $k \in \{1...n\}$ je také, že $a_k=0$ a súčasne $\forall l \in \{1...n\}$, $l>k$ platí $a_l=1$. Potom ak pre u,v ako čísla v binárnej sústave platí $u+1 \neq v \Leftrightarrow$ platí jedno z nasledujúcich tvrdení:

1) $\exists j \in \{1...n\}$, $j<k$ také, že $a_j=0$ a súčasne $b_j=1$

2) $\exists j \in \{1...n\}$, $j<k$ také, že $a_j=1$ a súčasne $b_j=0$

3) $b_k=0$

4) $\exists j \in \{1...n\}$, $j>k$ také, že $b_j=1$

Dôkaz Lemy:

$\Rightarrow$: Dokážeme obmenu, teda, keď sú splnené všetky predpoklady a tvrdenia 1) - 4) všetky neplatia, potom u+1=v.

Nech $u,v \in \{0,1\}^*$, $u=a_1...a_n$, $v=b_1...b_n$, $a_1= b_1=1$, $n \leq 2$, nech $i \in \{1..n\}$ také, že $a_i=0$. Nech $k \in \{1...n\}$ je také, že $a_k=0$ a súčasne $\forall l \in \{1...n\}$, $l>k$ platí $a_l=1$. Nech ďalej platí: $\forall j \in \{1...n\}$, $j<k$ platí $a_j=0 \Rightarrow b_j=0$

$\forall j \in \{1...n\}$, $j<k$ platí $a_j=1 \Rightarrow b_j=0$

$b_k=1$

$\forall j \in \{1...n\}$, $j>k$ je $b_j=0$

teda ak $u=a_1...a_{k-1}01^{n-k}$, potom $v=a_1...a_{k-1}10^{n-k}$ a teda pre u,v ako čísla v bin. sústave platí u+1=v.

$\Leftarrow$: Znova dokážeme obmenu, teda že ak platia predpoklady a u+1=v, tak neplatí ani jedno z tvrdení 1) - 4).

Nech $u=a_1...a_{k-1}01^{n-k}$ (že u môžeme písať v takomto tvare vyplýva z predpokladu lemy). Potom $u+1=v=a_1...a_{k-1}10^{n-k}$, ale tiež $v=b_1...b_n$, teda $b_1=a_1,...$, $b_{k-1}=a_{k-1}$, $b_k=1$, $b_{k+1}=0,...$, $b_n=0$. Z tohto vidíme, že 3) neplatí. Tiež platí $\forall j \in \{1...n\}$, $j>k$ je $b_j=0$ teda neplatí 4). 1) a 2) neplatia, lebo $\forall j \in \{1...n\}$, $j<k$ platí $a_j=b_j$.

Teraz prejdeme k samotnému dôkazu L(A)=L.

L⊆L(A): Nech $w = 1uc1v \in L$. Pre u uvažujeme niekoľko možností:

u ∈ {1}*: Najprv indukciou na |u| ukážeme že $(q_{211}, 1ucv, Z_0) \vdash^* (q_{211}, cv, Z_0 1^{|u|+1})$.

1° : $u=\varepsilon$ $(q_{211}, 1cv, Z_0) \vdash (q_{211}, cv, Z_01)$

2° : Nech tvrdenie platí pre všetky u také, že $|u|=k$ - toto je I.P.

Uvažujme slovo u1. $|u1|=k+1$. $(q_{211}, u1cv, Z_0) \vdash^*$ (I.P.) $\vdash^* (q_{211}, 1cv, Z_01^{k+1}) \vdash$

$\vdash (q_{211}, cv, Z_01^{k+2})$, keďže $k+2=|u1|+1$, tvrdenie platí.

Nech teraz $w=1uc1v$. Akceptačný výpočet v automate A začne nasledovne:

$(q_0, 1uc1v, Z_0) \vdash (q_{211}, 1uc1v, Z_0) \vdash^*$ (predošlý dôkaz) $\vdash^* (q_{211}, c1v, Z_01^{|u|+1})$.

Budeme teraz uvažovať všetky možnosti pre v:

1.) Nech $|v|<|u|+1$. Potom výpočet pokračuje takto:

$(q_{211}, c1v, Z_01^{|u|+1}) \vdash (q_{212}, 1v, Z_01^{|u|+1}) \vdash (q_{213}, v, Z_01^{|u|+1}) \vdash^{|v|} (q_{213}, \varepsilon, Z_01^{|u|+1-|v|})$

$\vdash (q_{215}, \varepsilon, Z_01^{|u|+1-|v|})$, keďže $|u|+1-|v|>0$ a $q_{215} \in F$.

2.) Nech $|v| \geq |u|+1$. Pre v môžu nastať dve možnosti:

a) $v=0^i1v'$, $i \in \{0,...,|u|\}$, $v' \in \{0,1\}^*$. Potom výpočet pokračuje takto:

$(q_{211}, c10^i1v', Z_01^{|u|+1}) \vdash (q_{212}, 10^i1v', Z_01^{|u|+1}) \vdash (q_{213}, 0^i1v', Z_01^{|u|+1}) \vdash^i$

$\vdash^i (q_{213}, 1v', Z_01^{|u|+1-i}) \vdash (q_{214}, v', Z_01^{|u|+1-i}) \vdash^* (q_{214}, \varepsilon, Z_01^{|u|+1-i})$, keďže

$|u|+1-i>0$ a $q_{214} \in F$.

b) $v=0^{|u|+1}av'$, $a \in \{0,1\}$, $v' \in \{0,1\}^*$. Potom výpočet pokračuje takto:

$(q_{211}, c10^{|u|+1}av', Z_01^{|u|+1}) \vdash (q_{212}, 10^{|u|+1}av', Z_01^{|u|+1}) \vdash (q_{213}, 0^{|u|+1}av', Z_01^{|u|+1})$

$\vdash^{|u|+1} (q_{213}, av', Z_0) \vdash (q_{216}, v', Z_0) \vdash^* (q_{216}, \varepsilon, Z_0)$, keďže $q_{216} \in F$.

<u>$u \notin \{1\}^*$:</u> Uvažujme slovo $w=1uc1v$. Rozlíšime dve možnosti:

1) Nech $|u| \neq |v|$. Keďže $u \notin \{1\}^*$, môžeme u napísať ako $u=1^i0u'$, $i \in N$, $u' \in \{0,1\}^*$.

Najprv indukciou na i ukážeme, že $(q_{211}, 1ucv, Z_0) \vdash^* (q_{221}, cv, Z_01^{|u|+1})$.

1° : Nech i=0. $u=0u'$, $u' \in \{0,1\}^*$.

$(q_{211}, 10u'cv, Z_0) \vdash (q_{211}, 0u'cv, Z_01) \vdash (q_{221}, u'cv, Z_011) \vdash^*$

$\vdash^* (q_{221}, cv, Z_0111^{|u'|})$, $111^{|u'|}=1^{|u'|+2}=1^{|u|+1}$.

2° : Nech tvrdenie platí pre i=k. Ukážeme, že platí aj pre i=k+1 (teda pre $u=1^{k+1}0u'$). Ak i=k+1, tak $|u|=k+2+|u'|$. Výpočet vyzerá takto:

$(q_{211}, 111^k0u'cv, Z_0) \vdash (q_{211}, 11^k0u'cv, Z_01) \vdash (q_{211}, 1^k0u'cv, Z_011) \vdash^*$ (I.P)

$\vdash^* (q_{221}, cv, Z_0111^{k+1+|u'|})$, $111^{k+1+|u'|}=1^{k+3+|u'|}=1^{|u|+1}$.

Ďalej pre v môžu nastať dve možnosti:

a) $|u|<|v|$. Výpočet pokračuje nasledovne:

$(q_{221}, c1v, Z_0 1^{|u|+1}) \vdash (q_{225}, 1v, Z_0 1^{|u|+1}) \vdash (q_{222}, v, Z_0 1^{|u|}) \vdash^* (q_{222}, av', Z_0) \vdash$
$\vdash (q_{224}, v', Z_0) \vdash^* (q_{224}, \varepsilon, Z_0)$, keďže $a \in \{0,1\}$, $v' \in \{0,1\}^*$ (do tejto konfigurácie sme sa dostali vďaka predpokladu, že $|u|<|v|$) a $q_{224} \in F$.

b) $|u|>|v|$. Výpočet pokračuje nasledovne:
$(q_{221}, c1v, Z_0 1^{|u|+1}) \vdash (q_{225}, 1v, Z_0 1^{|u|+1}) \vdash (q_{222}, v, Z_0 1^{|u|}) \vdash^* (q_{222}, \varepsilon, Z_0 1^{|u|-|v|}) \vdash$
$\vdash (q_{223}, \varepsilon, Z_0 1^{|u|-|v|})$, keďže $|u|-|v|>0$ a $q_{223} \in F$.

2) Nech $|u|=|v|$. Potom u, v spĺňajú predpoklady Lemy, a teda je pre ne splnené jedno z tvrdení Lemy. Uvážime teraz všetky štyri možnosti a ukážeme príslušné akceptačné výpočty:

a) Nech je splnená podmienka 1. Uvažujme najmenšie také j, pre ktoré daná podmienka platí. Potom:
$u=1u_1 0u_2 0u_3$, kde $u_1,u_3 \in \{0,1\}^*$, $u_2 \in \{1\}^*$, $|u_1|=j-2$.
$v=1v_1 1v_2$, kde $v_1,v_2 \in \{0,1\}^*$, $|v_1|=j-2$. Výpočet vyzerá nasledovne (z def. δ):
$(q_0, 1u_1 0u_2 0u_3 c1v_1 1v_2, Z_0) \vdash (q_1, u_1 0u_2 0u_3 c1v_1 1v_2, Z_0) \vdash^{j-2}$
$\vdash^{j-2} (q_1, 0u_2 0u_3 c1v_1 1v_2, Z_0 1^{j-2}) \vdash (q_{10}, u_2 0u_3 c1v_1 1v_2, Z_0 1^{j-2}) \vdash^*$
$\vdash^* (q_{10}, 0u_3 c1v_1 1v_2, Z_0 1^{j-2}) \vdash (q_{1011}, u_3 c1v_1 1v_2, Z_0 1^{j-2}) \vdash^*$
$\vdash^* (q_{1011}, c1v_1 1v_2, Z_0 1^{j-2}) \vdash (q_{1012}, 1v_1 1v_2, Z_0 1^{j-2}) \vdash (q_{1013}, v_1 1v_2, Z_0 1^{j-2}) \vdash^{j-2}$
$\vdash^{j-2} (q_{1013}, 1v_2, Z_0) \vdash (q_{1014}, v_2, Z_0) \vdash^* (q_{1014}, \varepsilon, Z_0)$, $q_{1014} \in F$.

b) Nech je splnená podmienka 2. Uvažujme najmenšie také j, pre ktoré daná podmienka platí. Potom:
$u=1u_1 1u_2 0u_3$, kde $u_1,u_3 \in \{0,1\}^*$, $u_2 \in \{1\}^*$, $|u_1|=j-2$.
$v=1v_1 0v_2$, kde $v_1,v_2 \in \{0,1\}^*$, $|v_1|=j-2$. Výpočet vyzerá nasledovne (z def. δ):
$(q_0, 1u_1 1u_2 0u_3 c1v_1 0v_2, Z_0) \vdash (q_1, u_1 1u_2 0u_3 c1v_1 0v_2, Z_0) \vdash^{j-2}$
$\vdash^{j-2} (q_1, 1u_2 0u_3 c1v_1 0v_2, Z_0 1^{j-2}) \vdash (q_{11}, u_2 0u_3 c1v_1 0v_2, Z_0 1^{j-2}) \vdash^*$
$\vdash^* (q_{11}, 0u_3 c1v_1 0v_2, Z_0 1^{j-2}) \vdash (q_{1111}, u_3 c1v_1 0v_2, Z_0 1^{j-2}) \vdash^*$
$\vdash^* (q_{1111}, c1v_1 0v_2, Z_0 1^{j-2}) \vdash (q_{1112}, 1v_1 0v_2, Z_0 1^{j-2}) \vdash (q_{1113}, v_1 0v_2, Z_0 1^{j-2}) \vdash^{j-2}$
$\vdash^{j-2} (q_{1113}, 0v_2, Z_0) \vdash (q_{1114}, v_2, Z_0) \vdash^* (q_{1114}, \varepsilon, Z_0)$, $q_{1114} \in F$.

c) Nech je splnená podmienka 3. Potom:
$u=1u_1 0u_2$, kde $u_1 \in \{0,1\}^*$, $u_2 \in \{1\}^*$, $|u_1|=k-2$.
$v=1v_1 0v_2$, kde $v_1,v_2 \in \{0,1\}^*$, $|v_1|=k-2$. Výpočet vyzerá nasledovne (z def. δ):
$(q_0, 1u_1 0u_2 c1v_1 0v_2, Z_0) \vdash (q_1, u_1 0u_2 c1v_1 0v_2, Z_0) \vdash^{k-2} (q_1, 0u_2 c1v_1 0v_2, Z_0 1^{k-2}) \vdash$

$\vdash (q_{10}, u_2c1v_10v_2, Z_01^{k-2}) \vdash^* (q_{10}, c1v_10v_2, Z_01^{k-2}) \vdash (q_{1001}, 1v_10v_2, Z_01^{k-2}) \vdash$

$\vdash (q_{1002}, v_10v_2, Z_01^{k-2}) \vdash^{k-2} (q_{1002}, 0v_2, Z_0) \vdash (q_{1003}, v_2, Z_0) \vdash^* (q_{1003}, \varepsilon, Z_0)$,

$q_{1003} \in F$.

d) Nech je splnená podmienka 4. Uvažujme najmenšie také j, pre ktoré daná podmienka platí. Potom:

$u=1u_10u_21u_3$, kde $u_1 \in \{0,1\}^*$, $u_2,u_3 \in \{1\}^*$, $|u_1|=k-2$, $|u_2|=j-1-k$.

$v=1v_11v_2$, kde $v_1,v_2 \in \{0,1\}^*$, $|v_1|=j-2$. Výpočet vyzerá nasledovne (z def. δ):

$(q_0, 1u_10u_21u_3c1v_11v_2, Z_0) \vdash (q_1, u_10u_21u_3c1v_11v_2, Z_0) \vdash^{k-2}$

$\vdash^{k-2} (q_1, 0u_21u_3c1v_11v_2, Z_01^{k-2}) \vdash (q_1, u_21u_3c1v_11v_2, Z_01^{k-1}) \vdash^{j-1-k}$

$\vdash^{j-1-k} (q_1, 1u_3c1v_11v_2, Z_01^{k-1+j-1-k}) \vdash (q_{11}, u_3c1v_11v_2, Z_01^{j-2}) \vdash^*$

$\vdash^* (q_{11}, c1v_11v_2, Z_01^{j-2}) \vdash (q_{1101}, 1v_11v_2, Z_01^{j-2}) \vdash (q_{1102}, v_11v_2, Z_01^{j-2}) \vdash^{j-2}$

$\vdash^{j-2} (q_{1102}, 1v_2, Z_0) \vdash (q_{1103}, v_2, Z_0) \vdash^* (q_{1103}, \varepsilon, Z_0)$, $q_{1103} \in F$.

$L(A) \subseteq L$: Majme $w \in L(A)$. Nech $w=a_1...a_n$. Pre w máme akceptačný výpočet v automate A tvaru $(q_0, a_1...a_n, Z_0) \vdash^* (q_F, \varepsilon, \gamma)$, $q_F \in F$, $\gamma \in \Gamma^*$. Najprv ukážeme, že $a_1=1$. Ak by $a_1=0$, pre výpočet by musel byť $(q_0, 0a_2...a_n, Z_0) \vdash (q_{211}, 0a_2...a_n, Z_0)$. Keďže $\delta(q_{211}, 0, Z_0) = \varnothing$, výpočet by nemohol byť akceptačný. Teda $a_1=1$. Teraz ukážeme, že $w=ucv$. Predpokladáme, že $w \neq ucv$, teda, že $w \in 1\{0,1\}^*$. Indukciou na $|w|$ ukážeme, že pre takéto slovo sa automat A dostane najďalej do jedného zo stavov z množiny $M=\{q_1, q_{10}, q_{1011}, q_{11}, q_{1111}, q_{211}, q_{221}\}$.

1° : Nech $|w|=1$, teda $w=1$. $(q_0, 1, Z_0) \vdash (q_1, \varepsilon, Z_0)$, $\delta(q_1, \varepsilon, Z_0) = \varnothing$

$(q_0, 1, Z_0) \vdash (q_{211}, 1, Z_0) \vdash (q_{211}, \varepsilon, Z_0)$, $\delta(q_{211}, \varepsilon, Z_0) = \varnothing$

2° : Nech $|w|=k$ a tvrdenie platí. Uvažujme slovo wa, $a \in \{0,1\}$, potom $|wa|=k+1$.

Symboly $\gamma,\gamma' \in \Gamma^*$, $x \in \Gamma$.

$(q_0, wa, Z_0) \vdash^* (I.P.) \vdash^* (q_1, a, \gamma)$,

ak $a=0 \Rightarrow (q_1, 0, \gamma) \vdash (q_1, \varepsilon, \gamma'x)$, $\delta(q_1, \varepsilon, x) = \varnothing$

$(q_1, 0, \gamma) \vdash (q_{10}, \varepsilon, \gamma'x)$, $\delta(q_{10}, \varepsilon, x) = \varnothing$

ak $a=1 \Rightarrow (q_1, 1, \gamma) \vdash (q_1, \varepsilon, \gamma'x)$, $\delta(q_1, \varepsilon, x) = \varnothing$

$(q_1, 1, \gamma) \vdash (q_{11}, \varepsilon, \gamma'x)$, $\delta(q_{11}, \varepsilon, x) = \varnothing$

$(q_0, wa, Z_0) \vdash^* (I.P.) \vdash^* (q_{10}, a, \gamma)$,

ak $a=0 \Rightarrow (q_{10}, 0, \gamma) \vdash (q_{1011}, \varepsilon, \gamma'x)$, $\delta(q_{1011}, \varepsilon, x) = \varnothing$

ak $a=1 \Rightarrow (q_{10}, 1, \gamma) \vdash (q_{10}, \varepsilon, \gamma'x)$, $\delta(q_{10}, \varepsilon, x) = \emptyset$

$(q_0, wa, Z_0) \vdash^* (I.P.) \vdash^* (q_{1011}, a, \gamma)$,
ak $a=0 \Rightarrow (q_{1011}, 0, \gamma) \vdash (q_{1011}, \varepsilon, \gamma'x)$, $\delta(q_{1011}, \varepsilon, x) = \emptyset$
ak $a=1 \Rightarrow (q_{1011}, 1, \gamma) \vdash (q_{1011}, \varepsilon, \gamma'x)$, $\delta(q_{1011}, \varepsilon, x) = \emptyset$

$(q_0, wa, Z_0) \vdash^* (I.P.) \vdash^* (q_{11}, a, \gamma)$,
ak $a=0 \Rightarrow (q_{11}, 0, \gamma) \vdash (q_{1111}, \varepsilon, \gamma'x)$, $\delta(q_{1111}, \varepsilon, x) = \emptyset$
ak $a=1 \Rightarrow (q_{11}, 1, \gamma) \vdash (q_{11}, \varepsilon, \gamma'x)$, $\delta(q_{11}, \varepsilon, x) = \emptyset$

$(q_0, wa, Z_0) \vdash^* (I.P.) \vdash^* (q_{1111}, a, \gamma)$,
ak $a=0 \Rightarrow (q_{1111}, 0, \gamma) \vdash (q_{1111}, \varepsilon, \gamma'x)$, $\delta(q_{1111}, \varepsilon, x) = \emptyset$
ak $a=1 \Rightarrow (q_{1111}, 1, \gamma) \vdash (q_{1111}, \varepsilon, \gamma'x)$, $\delta(q_{1111}, \varepsilon, x) = \emptyset$

$(q_0, wa, Z_0) \vdash^* (I.P.) \vdash^* (q_{211}, a, \gamma)$,
ak $a=0 \Rightarrow (q_{211}, 0, \gamma) \vdash (q_{221}, \varepsilon, \gamma'x)$, $\delta(q_{221}, \varepsilon, x) = \emptyset$
ak $a=1 \Rightarrow (q_{211}, 1, \gamma) \vdash (q_{211}, \varepsilon, \gamma'x)$, $\delta(q_{211}, \varepsilon, x) = \emptyset$

$(q_0, wa, Z_0) \vdash^* (I.P.) \vdash^* (q_{221}, a, \gamma)$,
ak $a=0 \Rightarrow (q_{221}, 0, \gamma) \vdash (q_{221}, \varepsilon, \gamma'x)$, $\delta(q_{221}, \varepsilon, x) = \emptyset$
ak $a=1 \Rightarrow (q_{221}, 1, \gamma) \vdash (q_{221}, \varepsilon, \gamma'x)$, $\delta(q_{221}, \varepsilon, x) = \emptyset$

Keďže $\forall q \in M\ q \notin F$, a tiež $\delta(q, c, x) \neq \emptyset$ pre nejaké $x \in \Gamma$, tak aby výpočet pre w mohol byť akceptačný, musí byť w=ucv. Taktiež už vieme, že u=1u', kde $u' \in \{0,1\}^*$.
Uvažujme teraz všetky možnosti pre akceptačný stav q_F v danom akceptačnom výpočte pre w. Ukážeme, že ak je tento koncový stav ľubovoľný z koncových stavov automatu A, potom musí nutne $w \in L$. Budeme predpokladať, že $u=1u_2...u_n$, a $v=v_1...v_m$.

1.) Nech $q_F = q_{214}$.
$(q_0, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_{211}, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_{211}, u_2...u_ncv_1...v_m, Z_01) \vdash^{n-1}$

$\vdash^{n-1} (q_{211}, cv_1...v_m, Z_0 1^n)$, teda $u \in 1\{1\}^*$ (ak by nebolo, skončíme v stave q_{221}, a z def. δ vidíme, že odtiaľ sa už do q_{214} nemôžeme dostať)

$(q_{211}, cv_1...v_m, Z_0 1^n) \vdash (q_{212}, v_1...v_m, Z_0 1^n)$, keďže $\delta(q_{212}, 0, 1) = \emptyset$ a

$\delta(q_{212}, 1, 1) = \{ (q_{213}, 1) \}$, je $v_1 = 1$.

$(q_{212}, 1v_2...v_m, Z_0 1^n) \vdash (q_{213}, v_2...v_m, Z_0 1^n) \vdash^{i-1} (q_{213}, 1v_{i+2}...v_m, Z_0 1^{n-i} 1) \vdash$

$\vdash (q_{214}, v_{i+2}...v_m, Z_0 1^{n-i} 1) \vdash^* (q_{214}, \varepsilon, Z_0 1^{n-i} 1)$. Z def. δ vyplýva $i \leq n-1$ a tiež vidíme, že $v_{i+1}=1$. Potom ale $ucv \in L$, lebo ak by nebolo, muselo by byť $v=10^n$, čo je spor s tým, že $v_{i+1}=1$.

2.) Nech $q_F = q_{215}$.

$(q_0, 1u_2...u_n cv_1...v_m, Z_0) \vdash (q_{211}, 1u_2...u_n cv_1...v_m, Z_0) \vdash (q_{211}, u_2...u_n cv_1...v_m, Z_0 1) \vdash^{n-1}$

$\vdash^{n-1} (q_{211}, cv_1...v_m, Z_0 1^n)$, teda $u \in 1\{1\}^*$ (ak by nebolo, skončíme v stave q_{221}, a z def. δ vidíme, že odtiaľ sa už do q_{215} nemôžeme dostať)

$(q_{211}, cv_1...v_m, Z_0 1^n) \vdash (q_{212}, v_1...v_m, Z_0 1^n)$, keďže $\delta(q_{212}, 0, 1) = \emptyset$ a

$\delta(q_{212}, 1, 1) = \{ (q_{213}, 1) \}$, je $v_1 = 1$.

$(q_{212}, 1v_2...v_m, Z_0 1^n) \vdash (q_{213}, v_2...v_m, Z_0 1^n) \vdash^{m-1} (q_{213}, \varepsilon, Z_0 1^{n-m} 1) \vdash (q_{215}, \varepsilon, Z_0 1^{n-m} 1)$. Z def. δ vyplýva $m \leq n$ a teda $|u| \geq |v|$. Potom ale $ucv \in L$.

3.) Nech $q_F = q_{216}$.

$(q_0, 1u_2...u_n cv_1...v_m, Z_0) \vdash (q_{211}, 1u_2...u_n cv_1...v_m, Z_0) \vdash (q_{211}, u_2...u_n cv_1...v_m, Z_0 1) \vdash^{n-1}$

$\vdash^{n-1} (q_{211}, cv_1...v_m, Z_0 1^n)$, teda $u \in 1\{1\}^*$ (ak by nebolo, skončíme v stave q_{221}, a z def. δ vidíme, že odtiaľ sa už do q_{215} nemôžeme dostať)

$(q_{211}, cv_1...v_m, Z_0 1^n) \vdash (q_{212}, v_1...v_m, Z_0 1^n)$, keďže $\delta(q_{212}, 0, 1) = \emptyset$ a

$\delta(q_{212}, 1, 1) = \{ (q_{213}, 1) \}$, je $v_1 = 1$.

$(q_{212}, 1v_2...v_m, Z_0 1^n) \vdash (q_{213}, v_2...v_m, Z_0 1^n) \vdash^{n} (q_{213}, v_{n+2}v_{n+3}...v_m, Z_0) \vdash$

$\vdash (q_{216}, v_{n+3}...v_m, Z_0) \vdash^* (q_{216}, \varepsilon, Z_0)$. Z def. δ vyplýva $v_{n+2} \in \{0,1\}$ a $m \geq n+2$ a teda $|u|+2 \leq |v|$. Potom ale $ucv \in L$.

4.) Nech $q_F = q_{223}$.

$(q_0, 1u_2...u_n cv_1...v_m, Z_0) \vdash (q_{211}, 1u_2...u_n cv_1...v_m, Z_0) \vdash (q_{211}, u_2...u_n cv_1...v_m, Z_0 1) \vdash^{i-2}$

$\vdash^{i-2} (q_{211}, u_i u_{i+1}...u_n cv_1...v_m, Z_0 1^{i-1})$, keďže sa musíme dostať do stavu q_{221}, lebo z iného sa už do q_{223} nedostaneme, musí byť pre $i \in \{1...n\}$ $u_i = 0$ a teda $u \notin 1\{1\}^*$.

$(q_{211}, 0u_{i+1}...u_ncv_1...v_m, Z_01^{i-1}) \vdash (q_{221}, u_{i+1}...u_ncv_1...v_m, Z_01^i) \vdash^{n-i}$

$\vdash^{n-i} (q_{221}, cv_1...v_m, Z_01^n) \vdash (q_{225}, v_1...v_m, Z_01^n)$, keďže $\delta(q_{225}, 0, 1) = \emptyset$ a

$\delta(q_{225}, 1, 1)=\{ (q_{222}, \varepsilon) \}$, je $v_1=1$.

$(q_{225}, 1v_2...v_m, Z_01^n) \vdash (q_{222}, v_2...v_m, Z_01^{n-1}) \vdash^{m-1} (q_{222}, \varepsilon, Z_01^{n-m}) \vdash (q_{223}, \varepsilon, Z_01^{n-m})$

Z def. δ vyplýva $m<n$ a teda $|u|>|v|$. Potom ale $ucv \in L$.

5.) Nech $q_F= q_{224}$.

$(q_0, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_{211}, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_{211}, u_2...u_ncv_1...v_m, Z_01) \vdash^{i-2}$

$\vdash^{i-2} (q_{211}, u_iu_{i+1}...u_ncv_1...v_m, Z_01^{i-1})$, keďže sa musíme dostať do stavu q_{221}, lebo z iného sa už do q_{223} nedostaneme, musí byť pre $i \in \{1...n\}$ $u_i=0$ a teda $u \notin 1\{1\}^*$.

$(q_{211}, 0u_{i+1}...u_ncv_1...v_m, Z_01^{i-1}) \vdash (q_{221}, u_{i+1}...u_ncv_1...v_m, Z_01^i) \vdash^{n-i}$

$\vdash^{n-i} (q_{221}, cv_1...v_m, Z_01^n) \vdash (q_{225}, v_1...v_m, Z_01^n)$, keďže $\delta(q_{225}, 0, 1) = \emptyset$ a

$\delta(q_{225}, 1, 1)=\{ (q_{222}, \varepsilon) \}$, je $v_1=1$.

$(q_{225}, 1v_2...v_m, Z_01^n) \vdash (q_{222}, v_2...v_m, Z_01^{n-1}) \vdash^{n-1} (q_{222}, v_{n+1}\, v_{n+2}...v_m, Z_0) \vdash$

$\vdash (q_{224}, v_{n+2}...v_m, Z_0) \vdash^* (q_{224}, \varepsilon, Z_0)$

Z def. δ vyplýva $v_{n+1} \in \{0,1\}$, $m>n$ a teda $|u|<|v|$. Keďže $u \notin 1\{1\}^*$, je $ucv \in L$.

6.) Nech $q_F= q_{1003}$.

$(q_0, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_1, u_2...u_ncv_1...v_m, Z_0)$, aby sme sa do stavu q_{1003} mohli dostať, musíme sa najprv dostať do q_{10}, čo z def. δ ide len vtedy, ak $\exists\, i \in \{2..n\}$, že $u_i=0$.

$(q_1, u_2...u_ncv_1...v_m, Z_0) \vdash^{i-2} (q_1, 0u_{i+1}...u_ncv_1...v_m, Z_01^{i-2}) \vdash$

$\vdash (q_{10}, u_{i+1}...u_ncv_1...v_m, Z_01^{i-2})$. Ďalej sa potrebujeme dostať do stavu q_{1001}, čo z def. δ znamená, že $\forall\, j \in \{i+1...n\}$ je $u_j=1$.

$(q_{10}, u_{i+1}...u_ncv_1...v_m, Z_01^{i-2}) \vdash^{n-i} (q_{10}, cv_1...v_m, Z_01^{i-2}) \vdash (q_{1001}, v_1...v_m, Z_01^{i-2})$, keďže $\delta(q_{1001}, 0, x) = \emptyset$ a $\delta(q_{1001}, 1, x) = \{ (q_{1002}, x) \}$ pre $\forall x \in \Gamma$, je $v_1=1$.

$(q_{1001}, 1v_2...v_m, Z_01^{i-2}) \vdash (q_{1002}, v_2...v_m, Z_01^{i-2}) \vdash^{i-2} (q_{1002}, v_i...v_m, Z_0) \vdash$

$\vdash (q_{1003}, v_{i+1}...v_m, Z_0) \vdash^* (q_{1003}, \varepsilon, Z_0)$. Z def. δ vyplýva $v_i=0$, $i \leq m$ a (keďže $i \in \{2..n\}$) $n \leq m$ a teda $|u| \leq |v|$. Ak $|u|=|v|$, sú splnené všetky predpoklady Lemy a tiež podmienka 3) Lemy (lebo za k z Lemy položíme i) a teda $u+1 \neq v \Rightarrow ucv \in L$. Ak $|u|<|v|$, vieme, že $u \notin 1\{1\}^*$, preto $ucv \in L$.

7.) Nech $q_F = q_{1014}$.

$(q_0, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_1, u_2...u_ncv_1...v_m, Z_0)$, aby sme sa do stavu q_{1014} mohli dostať, musíme sa najprv dostať do q_{10}, čo z def. δ ide len vtedy, ak $\exists\, i \in \{2..n\}$, že $u_i=0$.

$(q_1, u_2...u_ncv_1...v_m, Z_0) \vdash^{i-2} (q_1, 0u_{i+1}...u_ncv_1...v_m, Z_0 1^{i-2}) \vdash$

$\vdash (q_{10}, u_{i+1}...u_ncv_1...v_m, Z_0 1^{i-2})$. Ďalej sa potrebujeme dostať do stavu q_{1011}, čo z def. δ znamená, že $\exists\, j \in \{i+1...n\}$ je $u_j=0$.

$(q_{10}, u_{i+1}...u_ncv_1...v_m, Z_0 1^{i-2}) \vdash^{j-i-1} (q_{10}, 0u_{j+1}...u_ncv_1...v_m, Z_0 1^{i-2}) \vdash$

$\vdash (q_{1011}, u_{j+1}...u_ncv_1...v_m, Z_0 1^{i-2}) \vdash^{n-j} (q_{1011}, cv_1...v_m, Z_0 1^{i-2}) \vdash (q_{1012}, v_1...v_m, Z_0 1^{i-2})$, keďže $\delta(q_{1012}, 0, x) = \emptyset$ a $\delta(q_{1012}, 1, x) = \{ (q_{1013}, x) \}$ pre $\forall x \in \Gamma$, je $v_1=1$.

$(q_{1012}, 1v_2...v_m, Z_0 1^{i-2}) \vdash (q_{1013}, v_2...v_m, Z_0 1^{i-2}) \vdash^{i-2} (q_{1013}, v_i...v_m, Z_0) \vdash$

$\vdash (q_{1014}, v_{i+1}...v_m, Z_0) \vdash^* (q_{1014}, \varepsilon, Z_0)$. Z def. δ vyplýva $v_i=1$, $i \le m$ a (keďže $i \in \{2..n\}$) $n \le m$ a teda $|u| \le |v|$. Ak $|u|=|v|$, sú splnené všetky predpoklady Lemy a tiež podmienka 1) Lemy a teda $u+1 \ne v \Rightarrow ucv \in L$. Ak $|u|<|v|$, vieme, že $u \notin 1\{1\}^*$, preto $ucv \in L$.

8.) Nech $q_F = q_{1103}$.

$(q_0, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_1, u_2...u_ncv_1...v_m, Z_0)$, aby sme sa do stavu q_{1103} mohli dostať, musíme sa najprv dostať do q_{11}, čo z def. δ ide len vtedy, ak $\exists\, i \in \{2..n\}$, že $u_i=1$.

$(q_1, u_2...u_ncv_1...v_m, Z_0) \vdash^{i-2} (q_1, 1u_{i+1}...u_ncv_1...v_m, Z_0 1^{i-2}) \vdash$

$\vdash (q_{11}, u_{i+1}...u_ncv_1...v_m, Z_0 1^{i-2})$. Ďalej sa potrebujeme dostať do stavu q_{1101}, čo z def. δ znamená, že $\forall\, j \in \{i+1...n\}$ je $u_j=1$.

$(q_{11}, u_{i+1}...u_ncv_1...v_m, Z_0 1^{i-2}) \vdash^{n-i} (q_{11}, cv_1...v_m, Z_0 1^{i-2}) \vdash (q_{1101}, v_1...v_m, Z_0 1^{i-2})$, keďže $\delta(q_{1101}, 0, x) = \emptyset$ a $\delta(q_{1101}, 1, x) = \{ (q_{1102}, x) \}$ pre $\forall x \in \Gamma$, je $v_1=1$.

$(q_{1101}, 1v_2...v_m, Z_0 1^{i-2}) \vdash (q_{1102}, v_2...v_m, Z_0 1^{i-2}) \vdash^{i-2} (q_{1102}, v_i...v_m, Z_0) \vdash$

$\vdash (q_{1103}, v_{i+1}...v_m, Z_0) \vdash^* (q_{1103}, \varepsilon, Z_0)$. Z def. δ vyplýva $v_i=1$, $i \le m$ a (keďže $i \in \{2..n\}$) $n \le m$ a teda $|u| \le |v|$. Ak $|u|=|v|$, máme 2 možnosti:

a) $u \in 1\{1\}^*$, potom $ucv \in L$, lebo $|u|=|v|$.

b) $u \notin 1\{1\}^*$, potom sú splnené všetky predpoklady Lemy a tiež podmienka 4) Lemy (ak $u \notin 1\{1\}^*$, potom $\exists\, j \in \{1...n\}$ že $u_j=0$. Za k teraz vezmeme najväčšie také

j. Potom evidentne i>k, v_i=1 a teda podmienka 4) je splnená) a teda u+1≠v ⇒ ucv∈L.

Ak |u|<|v|, a u∉1{1}*, evidentne ucv∈L. Ak u∈1{1}* potom keďže v_i=1, i≥2, tak ucv∈L.

9.) Nech $q_F = q_{1114}$.

$(q_0, 1u_2...u_ncv_1...v_m, Z_0) \vdash (q_1, u_2...u_ncv_1...v_m, Z_0)$, aby sme sa do stavu q_{1114} mohli dostať, musíme sa najprv dostať do q_{11}, čo z def. δ ide len vtedy, ak ∃ i∈{2..n}, že u_i=1.

$(q_1, u_2...u_ncv_1...v_m, Z_0) \vdash^{i-2} (q_1, 1u_{i+1}...u_ncv_1...v_m, Z_01^{i-2}) \vdash$

$\vdash (q_{11}, u_{i+1}...u_ncv_1...v_m, Z_01^{i-2})$. Ďalej sa potrebujeme dostať do stavu q_{1111}, čo z def. δ znamená, že ∃ j∈{i+1...n} je u_j=0.

$(q_{11}, u_{i+1}...u_ncv_1...v_m, Z_01^{i-2}) \vdash^{j-i-1} (q_{11}, 0u_{j+1}...u_ncv_1...v_m, Z_01^{i-2}) \vdash$

$\vdash (q_{1111}, u_{j+1}...u_ncv_1...v_m, Z_01^{i-2}) \vdash^{n-j} (q_{1111}, cv_1...v_m, Z_01^{i-2}) \vdash (q_{1112}, v_1...v_m, Z_01^{i-2})$, keďže $\delta(q_{1112}, 0, x) = \emptyset$ a $\delta(q_{1112}, 1, x) = \{ (q_{1113}, x) \}$ pre ∀x∈Γ, je v_1=1.

$(q_{1112}, 1v_2...v_m, Z_01^{i-2}) \vdash (q_{1113}, v_2...v_m, Z_01^{i-2}) \vdash^{i-2} (q_{1113}, v_i...v_m, Z_0) \vdash$

$\vdash (q_{1114}, v_{i+1}...v_m, Z_0) \vdash^* (q_{1114}, \varepsilon, Z_0)$. Z def. δ vyplýva v_i=0, i≤m a (keďže i∈{2..n}) n≤m a teda |u|≤|v|. Ak |u|=|v|, sú splnené všetky predpoklady Lemy a tiež podmienka 2) Lemy a teda u+1≠v ⇒ ucv∈L. Ak |u|<|v|, vieme, že u∉1{1}*, preto ucv∈L.

Z tohto teda vyplýva, že nech sa dostaneme do ktoréhokoľvek koncového stavu automatu A, akceptované slovo vždy musí byť z L.